1) $X_{n+1} = 5X_n + 11 \pmod{17}$

$x_0 = 2$

$x_1 = 5 \cdot 2 + 11 \pmod{17} = 21 \bmod 17 = 4$

$x_2 = 5 \cdot 4 + 11 \pmod{17} = 31 \bmod 17 = 14$

$x_3 = 5 \cdot 14 + 11 \pmod{17} = 81 \bmod 17 = 13$

$x_4 = 5 \cdot 13 + 11 \pmod{17} = 76 \bmod 17 = 8$

~~(Se convierte en una)~~

$x_5 = 5 \cdot 8 + 11 \pmod{17} = 51 \bmod 17 = 0$

$x_6 = 5 \cdot 0 + 11 \pmod{17} = 11 \bmod 17 = 11$

$x_7 = 5 \cdot 11 + 11 \pmod{17} = 66 \bmod 17 = 15$

$x_8 = 5 \cdot 15 + 11 \pmod{17} = 86 \bmod 17 = 1$

$x_9 = 5 \cdot 1 + 11 \pmod{17} = 16 \bmod 17 = 16$

$x_{10} = 5 \cdot 16 + 11 \pmod{17} = 91 \bmod 17 = 6$

$x_{11} = 5 \cdot 6 + 11 \pmod{17} = 41 \bmod 17 = 7$

$x_{12} = 5 \cdot 7 + 11 \pmod{17} = 46 \bmod 17 = 12$

$x_{13} = 5 \cdot 12 + 11 \pmod{17} = 71 \bmod 17 = 3$

$x_{14} = 5 \cdot 3 + 11 \pmod{17} = 26 \bmod 17 = 9$

$x_{15} = 5 \cdot 9 + 11 \pmod{17} = 56 \bmod 17 = 5$

$x_{16} = 5 \cdot 5 + 11 \pmod{17} = 36 \bmod 17 = 2$

Es cíclica.

2) $\left\{ \begin{array}{l} -2x \equiv 8 \pmod{11} \\ 4x \equiv -3 \pmod{7} \\ 5x \equiv 10 \pmod{12} \\ 7x \equiv 6 \pmod{16} \\ 3x \equiv 10 \pmod{14} \end{array} \right. \equiv \left(\left\{ \frac{-2x - 8}{11} = 0 \right. \right)$

2) Ver final del documento.

3) $S(n,n) = S(n,1) = 1$ $\qquad n = 5.$

$S(5,5) = 1$ //

$S(5,4) = S(4,3) + 4\,\overset{1}{S(4,4)} = 10$ //

$S(5,3) = S(4,2) + 3\,S(4,3) = 25$ //

$\quad \hookrightarrow S(4,3) = S(3,2) + 3\,S(3,3) = 6$

$\quad S(3,3) = 1$

$S(5,2) = \overset{1}{S(4,1)} + 2\,S(4,2) = 15$ //

$\quad \hookrightarrow S(4,2) = \overset{1}{S(3,1)} + 2\,S(3,2) = 7$

$\quad S(3,2) = \overset{1}{S(2,1)} + 2\,S(2,2) = 3$

$\quad S(2,2) = 1$

$S(5,1) = 1$ //

4) a) $\sum_{i=0}^{n} \binom{n}{i} = 2^n$

Caso base: $n = 0 \quad \sum_{i=0}^{0} \binom{0}{i} = \binom{0}{0} = 2^0 = 1$ ✓

Caso recursivo: $\sum_{i=0}^{n+1} \binom{n+1}{i} = 2^{n+1}$

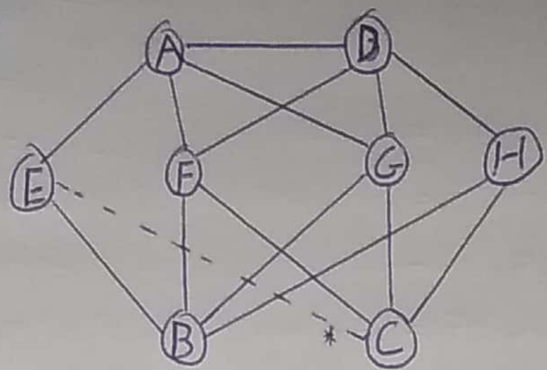
$$\binom{n+1}{i} = \binom{n}{i-1} + \binom{n}{i} = \frac{n!}{(i-1)!\,(n-(i-1))!} + \frac{n!}{i!\,(n-i)!} =$$

$$\frac{n!\,i}{i!\,(n-i)!\,(n+1-i)!} + \frac{n!}{i!\,(n-i)!} = \frac{(n+1)!}{i!\,(n+1-i)!}$$

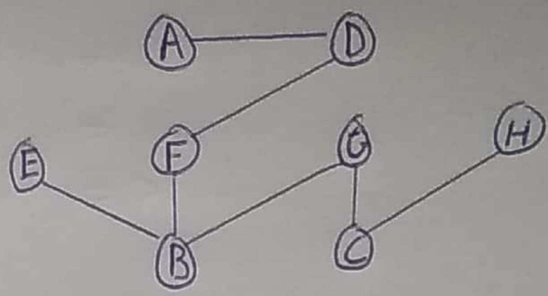
Con esto se demuestra que cualquier término de $\sum_{i=0}^{n} \binom{n}{i}$ está dos veces en $\sum_{i=0}^{n+1} \binom{n+1}{i}$. ✓

b) $\text{card}(A) = n \rightarrow \text{card}(P(A)) = 2^n.$

5) a)



* Arista adicional del apartado c.



b) No tiene un ciclo euleriano, pero si hamiltoniano (AEBFCHDGA)

c) Con la arista adicional el grafo se mantiene igual. Sigue sin presentar un ciclo euleriano y presenta el mismo ciclo hamiltoniano.

2)

$$\begin{cases} -2x \equiv 8 \mod 11 \rightarrow x \equiv 7 \mod 11 \\ 4x \equiv -3 \mod 7 \rightarrow x \equiv 1 \mod 7 \\ 5x \equiv 10 \mod 12 \rightarrow x \equiv 2 \mod 12 \\ 7x \equiv 6 \mod 16 \rightarrow x \equiv 10 \mod 16 \\ 3x \equiv 10 \mod 14 \rightarrow x \equiv 8 \mod 14 \end{cases}$$

$x = 7 + 11A = 29 + 77B = 722 + 924C = 2570 + 14784D = 2570$ //

$7 + 11A \equiv 1 \mod 7 \rightarrow A \equiv 2 \mod 7 \rightarrow A = 2 + 7B$

$29 + 77B \equiv 2 \mod 12 \rightarrow B \equiv 9 \mod 12 \rightarrow B = 9 + 12C$

$722 + 924C \equiv 10 \mod 16 \rightarrow C \equiv 2 \mod 16 \rightarrow C = 2 + 16D$

$2570 + 14784D \equiv 8 \mod 14 \rightarrow 0D \equiv 0 \mod 14 \rightarrow D = 0 + 0E$